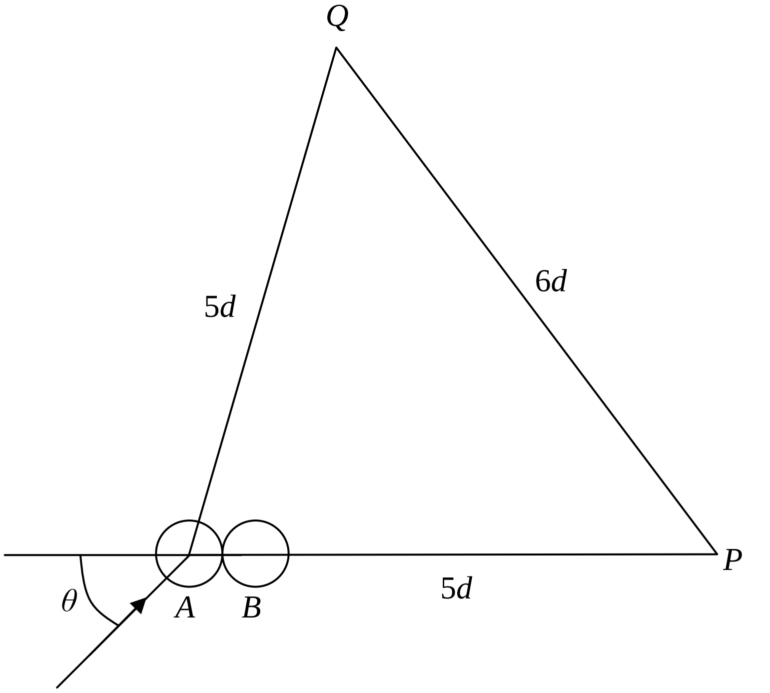
M4 – ELASTIC COLLISIONS IN TWO DIMENSIONS

Question:



A small smooth sphere *A* moving with speed *u* on a horizontal table collides with another identical sphere *B* which is at rest on the table. The direction of motion of *A* before impact makes an angle *θ*, where $\tan\theta = \frac{3}{2}$, with the line of centres of *A* and *B*. The coefficient of restitution between the two spheres is $\frac{1}{8}$. Two points *P* and *Q* on the table are each at a distance 5*d* from the position of the centre of *A* at the moment of impact and the distance *PQ* is 6*d* as shown in the diagram. After the impact *B* moves towards *P*.

(a) Find the velocity of *A* after impact.

(b) Show that *A* moves towards *Q*.

[*Edexcel*]

Solution:

Where *X* is the horizontal component and *Y* is the vertical component, before the impact:

$$X_A = u\cos\theta = \frac{2u}{\sqrt{13}} \qquad Y_A = u\sin\theta = \frac{3u}{\sqrt{13}}$$

$$X_B = 0 \qquad Y_B = 0$$

After the impact:

$$X_A = v_1 \qquad Y_A = u\sin\theta = \frac{3u}{\sqrt{13}}$$

$$X_B = v_2 \qquad Y_B = 0$$

By conservation of momentum:

$$mu\cos\theta = mv_1 + mv_2$$
$$v_1 + v_2 = u\cos\theta$$

By Newton's law of restitution:

$$v_2 - v_1 = \tfrac{1}{8}u\cos\theta$$

Now subtracting:

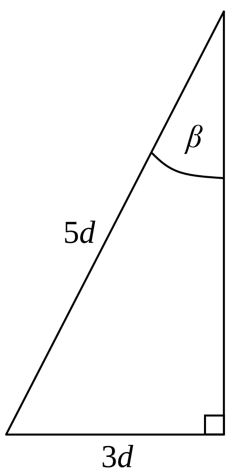
$$2v_1 = \tfrac{7}{8}u\cos\theta$$
$$v_1 = \tfrac{7}{16}u\cos\theta = \frac{7u}{8\sqrt{13}}$$
$$\therefore \quad v_B = \sqrt{\left(\frac{7u}{8\sqrt{13}}\right)^2 + \left(\frac{3u}{\sqrt{13}}\right)^2} = \frac{25u}{8\sqrt{13}}$$

It asking for the velocity, not just the speed, so we need to find the direction also:

$$\tan\alpha = \frac{\frac{3u}{\sqrt{13}}}{\frac{7u}{8\sqrt{13}}} = \tfrac{24}{7}$$
$$\alpha = \arctan\left(\tfrac{24}{7}\right)$$

Therefore A is travelling at a speed of $\dfrac{25u}{8\sqrt{13}}$ at an angle of $\arctan\left(\tfrac{24}{7}\right)$ to the line of centres.

For the next part we need to find the angle $P\hat{A}Q$ of the triangle and show that it is equal to α. Since the triangle is isosceles, it can be split into two equal right-angled triangles:



$$P\hat{A}Q = 2\beta$$
$$\sin\beta = \tfrac{3}{5} \quad \Rightarrow \quad \tan\beta = \tfrac{3}{4}$$

By the double-angle formula:

$$\tan(2\beta) = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{\frac{3}{2}}{1-\frac{9}{16}} = \frac{\frac{3}{2}}{\frac{7}{16}} = \tfrac{24}{7}$$
$$\therefore \quad 2\beta = \alpha$$