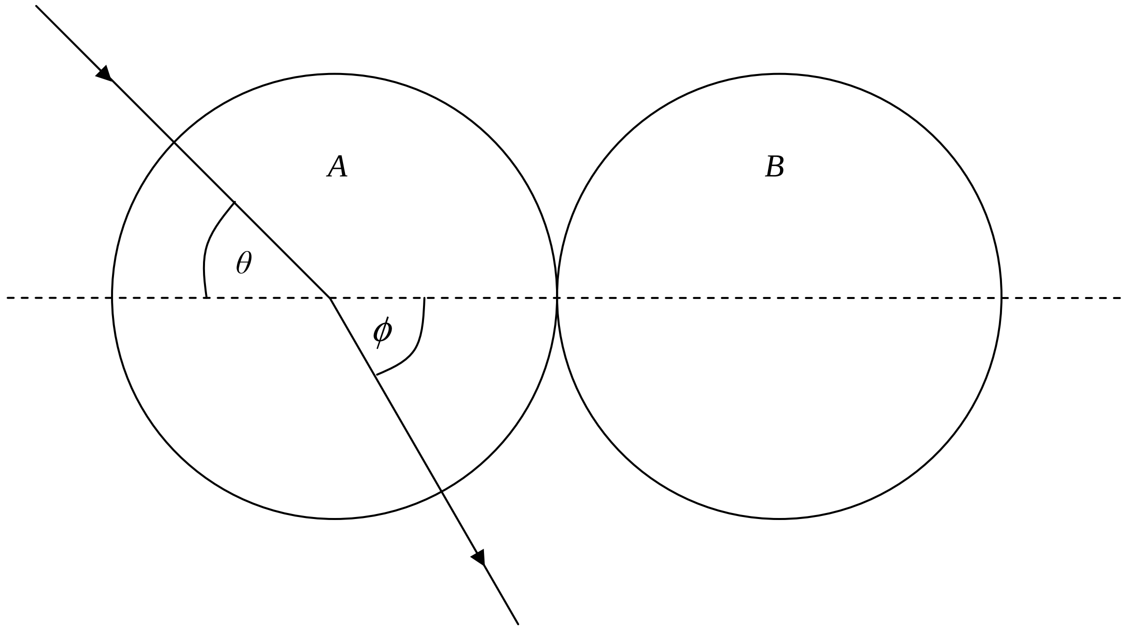
M4 – ELASTIC COLLISIONS IN TWO DIMENSIONS

Question:

A smooth uniform sphere *A* travelling along a smooth horizontal plane collides with a second smooth uniform sp here *B* of the same mass and radius which is at rest on the plane. At the moment of impact the direction of motion of A makes an angle θ with the line joining the centres of the spheres. Immediately after the impact the direction of motion of *A* makes an angle ϕ with the line joining the centres of the spheres, as shown below. The coefficient of restitution between the spheres is *e*, *e* > 1.



(a) Show that $\tan\phi = \dfrac{2\tan\theta}{1-e}$.

(b) Hence show that *A* is deflected by the impact through an angle δ, where

$$\tan\delta = \frac{(1+e)\tan\theta}{1-e+2\tan^2\theta}$$

[*Edexcel*]

Solution:

(a) Where *X* is the horizontal component and *Y* is the vertical component, before the impact:

$$X_A = u\cos\theta \qquad Y_A = u\sin\theta$$
$$X_B = 0 \qquad Y_B = 0$$

After the impact:

$$X_A = v_1 \qquad Y_A = u\sin\theta$$
$$X_B = v_2 \qquad Y_B = 0$$

So: $\tan\phi = \dfrac{u\sin\theta}{v_1}$

By conservation of momentum:

$$mu\cos\theta = mv_1 + mv_2$$
$$v_1 + v_2 = u\cos\theta$$

By Newton's law of restitution:

$$v_2 - v_1 = eu\cos\theta$$

Now subtracting:

$$2v_1 = u(1-e)\cos\theta$$
$$v_1 = \tfrac{1}{2}u(1-e)\cos\theta$$

Therefore:

$$\tan\phi = \frac{u\sin\theta}{\tfrac{1}{2}u(1-e)\cos\theta} = \frac{\tan\theta}{\tfrac{1}{2}(1-e)} = \frac{2\tan\theta}{(1-e)}$$

(b) Since $\delta = \phi - \theta$ we can use the addition/subtraction formula:

$$\tan\delta = \frac{\tan\phi - \tan\theta}{1+\tan\phi\tan\theta} = \frac{\dfrac{2\tan\theta}{1-e} - \tan\theta}{1+\dfrac{2\tan^2\theta}{1-e}}$$

$$\tan\delta = \frac{2\tan\theta - (1-e)\tan\theta}{1-e+2\tan^2\theta} = \frac{2\tan\theta - \tan\theta + e\tan\theta}{1-e+2\tan^2\theta}$$

$$\therefore \quad \tan\delta = \frac{(1+e)\tan\theta}{1-e+2\tan^2\theta}$$