

M4 – ELASTIC COLLISIONS IN TWO DIMENSIONS

Question:

A uniform smooth sphere A of mass m is moving with speed u on a horizontal table when it collides with a stationary uniform sphere B of the same size as A and of mass $2m$. The direction of motion of A immediately before the impact makes an angle α with the line of centres of the spheres. The coefficient of restitution between the spheres is $\frac{2}{3}$.

- (a) Find the speed of B immediately after the impact.

Given that the speed of A immediately after the impact is $\frac{1}{3}u$,

- (b) show that $\sin^2 \alpha = \frac{1}{10}$.

[Edexcel]

Solution:

- (a) Where X is the horizontal component and Y is the vertical component, before impact:

$$\begin{aligned} X_A &= u \cos \alpha & Y_A &= u \sin \alpha \\ X_B &= 0 & Y_B &= 0 \end{aligned}$$

$$\text{After impact: } \begin{aligned} X_A &= v_1 & Y_A &= u \sin \alpha \\ X_B &= v_2 & Y_B &= 0 \end{aligned}$$

$$\begin{aligned} \text{By conservation of momentum:} & & mu \cos \alpha &= mv_1 + 2mv_2 \\ & & v_1 + 2v_2 &= u \cos \alpha \end{aligned}$$

$$\text{By Newton's law of restitution: } v_2 - v_1 = eu \cos \alpha = \frac{2}{3}u \cos \alpha$$

$$\begin{aligned} \text{Now adding:} & & 3v_2 &= \frac{5}{3}u \cos \alpha \\ & & v_2 &= \frac{5}{9}u \cos \alpha \end{aligned}$$

- (b) Putting this into the CoM equation:
- $$\begin{aligned} v_1 + \frac{10}{9}u \cos \alpha &= u \cos \alpha \\ v_1 &= -\frac{1}{9}u \cos \alpha \end{aligned}$$

Therefore by Pythagoras:

$$\begin{aligned} \left(\frac{1}{3}u\right)^2 &= (v_1)^2 + u^2 \sin^2 \alpha \\ \frac{1}{9}u^2 &= \frac{1}{81}u^2 \cos^2 \alpha + u^2 \sin^2 \alpha \\ \frac{1}{9} &= \frac{1}{81}(1 - \sin^2 \alpha) + \sin^2 \alpha = \frac{1}{81} - \frac{1}{81}\sin^2 \alpha + \sin^2 \alpha \\ \frac{8}{81} &= \frac{80}{81}\sin^2 \alpha \\ 8 &= 80\sin^2 \alpha \\ \sin^2 \alpha &= \frac{8}{80} = \frac{1}{10} \end{aligned}$$