M4 – RELATIVE MOTION

Question:

Fishing boat A is moving with constant velocity of magnitude 10 m s^{-1} in the direction 060°. Fishing boat B is moving with constant velocity of magnitude 16 m s^{-1} due North. Show that the velocity of A relative to B is of magnitude 14 m s^{-1} and calculate the direction of this velocity, giving your answer as a bearing to the nearest tenth of a degree.

At noon, A is 5000m due West of B and T seconds later the distance between A and B is least. Calculate the value of T to the nearest integer and determine the least distance between A and B, giving your answer to the nearest ten metres.

Given that visibility is limited to 5000m, state the time, in seconds, for which the boats remain in visual contact.

[*Edexcel*]

Solution:

$$\mathbf{v}_A = (10\cos 30°)\mathbf{i} + (10\sin 30°)\mathbf{j}$$
$$\mathbf{v}_B = 16\mathbf{j}$$

$${}_A\mathbf{v}_B = (10\cos 30°)\mathbf{i} + (10\sin 30° - 16)\mathbf{j}$$
$$\left|{}_A\mathbf{v}_B\right| = \sqrt{(10\cos 30°)^2 + (10\sin 30° - 16)^2}$$
$$\left|{}_A\mathbf{v}_B\right| = 14 \text{ m s}^{-1}$$

$$\tan\theta = \frac{-(10\sin 30° - 16)}{10\cos 30°}$$
$$\theta = 51.8°$$
$$51.8° + 90° = 141.8° \text{ from North (1 d.p.)}$$

For the next part, we use the rule that ${}_A\mathbf{r}_B \cdot {}_A\mathbf{v}_B = 0$ when A and B are closest:

$${}_A\mathbf{v}_B = 5\sqrt{3}\mathbf{i} - 11\mathbf{j}$$
$${}_A\mathbf{r}_B = \left(5\sqrt{3}t - 5000\right)\mathbf{i} - 11t\mathbf{j}$$
$${}_A\mathbf{r}_B \cdot {}_A\mathbf{v}_B = \left(75T - 25000\sqrt{3}\right) + 121T = 0$$
$$196T = 25000\sqrt{3}$$
$$T = 221 \text{ secs}$$

Now to find the distance we put this value back into ${}_A\mathbf{r}_B$:

$${}_A\mathbf{r}_B = \left(1105\sqrt{3} - 5000\right)\mathbf{i} - 2431\mathbf{j}$$
$$\left|{}_A\mathbf{r}_B\right| = \sqrt{\left(1105\sqrt{3} - 5000\right)^2 + 2431^2} = 3930 \text{ m (3 s.f.)}$$

The time for which the boats remain in visual contact is $2T$ = 442 seconds (the boat takes 221 seconds to move from 5000m away to 3930m away, and since it is moving at constant velocity, it will take another 221 seconds to be 5000m away again).