

P5 – DIFFERENTIATION

Question:

Given that $y = \arcsin \frac{x}{a}$, where a is a constant, show that

$$\frac{d^2y}{dx^2} - x \left(\frac{dy}{dx} \right)^3 = 0$$

[Edexcel]

Solution:

$$\frac{dy}{dx} = \frac{1}{\sqrt{a^2 - x^2}} = (a^2 - x^2)^{-\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{2} \times -2x(a^2 - x^2)^{-\frac{3}{2}} = x(a^2 - x^2)^{-\frac{3}{2}}$$

$$\text{But } \left(\frac{dy}{dx} \right)^3 = (a^2 - x^2)^{-\frac{3}{2}}$$

$$\therefore \frac{d^2y}{dx^2} - x \left(\frac{dy}{dx} \right)^3 = 0$$

Q.E.D.