P6 – MATRICES

Question:

Given that $\mathbf{A}=\begin{pmatrix}4 & -3 & 1\\ 2 & 1 & -4\\ 1 & 2 & -2\end{pmatrix}$, find $\mathbf{A}^{-1}$.

[*Edexcel*]

Solution:

First find $\text{Det}\ \mathbf{A}=4\begin{vmatrix}1 & -4\\ 2 & -2\end{vmatrix}-(-3)\begin{vmatrix}2 & -4\\ 1 & -2\end{vmatrix}+1\begin{vmatrix}2 & 1\\ 1 & 2\end{vmatrix}=(4\times 6)-(-3\times 0)+(1\times 3)=27$

Now replace every element in the matrix by its minor:

$$\text{Minor}\ \mathbf{A}=\begin{pmatrix}6 & 0 & 3\\ 4 & -9 & 11\\ 11 & -18 & 10\end{pmatrix}$$

Now use the alternating law of signs $\begin{pmatrix}+ & - & +\\ - & + & -\\ + & - & +\end{pmatrix}$ to form the matrix of cofactors:

$$\text{Cofactor}\ \mathbf{A}=\begin{pmatrix}6 & 0 & 3\\ -4 & -9 & -11\\ 11 & 18 & 10\end{pmatrix}$$

Now transpose the matrix of cofactors:

$$(\text{Cofactor}\ \mathbf{A})^{\mathrm{T}}=\begin{pmatrix}6 & -4 & 11\\ 0 & -9 & 18\\ 3 & -11 & 10\end{pmatrix}$$

Finally, divide the matrix of cofactors by the determinant of the original matrix, and you have the inverse:

$$\mathbf{A}^{-1}=\tfrac{1}{27}\begin{pmatrix}6 & -4 & 11\\ 0 & -9 & 18\\ 3 & -11 & 10\end{pmatrix}$$